Key

Mean Value Theorem for Integrals Worksheet

For each problem, find the average value of the function over the given interval.

1. $f(x) = -x^2 - 2x + 5;\ [-4,0]$

Area under Curve = Area of Rect.

$$\int_{-4}^{0}(-x^2-2x+5)\,dx = (0-{}^-4)f(c)$$

$$\left[-\frac{x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}} + 5x\right]_{-4}^{0} = 4f(c)$$

$$(0)-\left(+\frac{64}{3}-16-20\right) = 4f(c)$$

$$-\frac{64}{3}+36 = 4f(c)$$

$$\frac{-64+108}{3} = 4f(c)$$

$$\frac{1}{\cancel{4}}\left(\frac{\cancel{44}^{11}}{3}\right) = (\cancel{4}f(c))\frac{1}{\cancel{4}} \Rightarrow \boxed{\frac{11}{3} = f(c)}$$

2. $f(x) = -x^4 + 2x^2 + 4;\ [-2,1]$

$$A_c = A_r$$

$$\int_{-2}^{1}(-x^4+2x^2+4)\,dx = (1-{}^-2)f(c)$$

$$\left[-\frac{x^5}{5} + \frac{2x^3}{3} + 4x\right]_{-2}^{1} = 3f(c)$$

$$\left(-\frac{1}{5}+\frac{2}{3}+4\right)-\left(\frac{32}{5}-\frac{16}{3}-8\right) = 3f(c)$$

$$-\frac{33}{5}+\frac{18}{3}+12 = 3f(c)$$

$$-\frac{33}{5}+18 = 3f(c)$$

$$\frac{-33+90}{5} = 3f(c)$$

$$\frac{1}{\cancel{3}}\left(\frac{\cancel{57}^{19}}{5}\right) = (\cancel{3}f(c))\frac{1}{\cancel{3}}$$

$$\boxed{\frac{19}{5} = f(c)}$$

3. $f(x) = \frac{4}{x^2};\ [-4,-2]$

$f(x) = 4x^{-2}$

$$A_c = A_R$$

$$\int_{-4}^{-2}(4x^{-2})\,dx = (-2-{}^-4)f(c)$$

$$\left[\frac{4\cdot x^{-1}}{-1}\right]_{-4}^{-2} = 2f(c)$$

$$\left[\frac{-4}{x}\right]_{-4}^{-2} = 2f(c)$$

$$2-1 = 2f(c)$$

$$1 = 2f(c)$$

$$\boxed{\frac{1}{2} = f(c)}$$